Study of Gaussian beam propagation using lenses of different focal length

Purpose

The goal of this experiment is to understand the irradiance profile of a Gaussian laser beam and to identify how the beam profile changes as it is focused using a lens

Background

Gaussian Beam: Beam waist and divergence

The He-Ne laser in the lab oscillates in the TEM_{00} mode. TEM means transverse electromagnetic mode and '00' refers to the fundamental mode. The beam produced by this oscillation is a Gaussian beam, meaning that the wave fronts of constant phase are spherical and thus cause the beam to diverge. The beam diameter, $2w$, increases as the beam propagates down the z-axis. The narrowest portion of the beam is called the beam waist and it has a diameter $2w_0$, as shown in Figure 1.

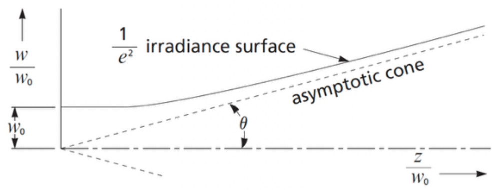

Figure 1. *Gaussian beam profile.*

For a Gaussian beam the beam diameter changes as a function of distance z, where z is measured as the distance from the beam waist $2w_0$. Also, a Gaussian beam has both a cylindrical and a spherical character which comes naturally out of the solution to the wave equation in cylindrical coordinates. Concentrating on its cylindrical character, the radius of the cylinder $w(z)$ at a distance z away from the beam waist is given by:

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_R}\right)^2} \quad (1)$$

where, the Rayleigh length, z_R is given by:

$$z_R = \frac{\pi w_0^2}{\lambda} \quad (2)$$

λ is the wavelength of light.

Considering the spherical character of the beam, with paraxial approximation, the half divergence angle θ (as shown in Figure 1) of the beam is given by

$$\theta = \frac{\lambda}{\pi w_0} \quad (3)$$

Note that to calculate the radius $w(z)$ and the divergence θ of a Gaussian laser beam, we need to know the minimum waist radius w_0. However, the beam waist of an unfocused laser beam typically occurs inside the laser's casing, so we cannot measure it directly. By using the Eq. 1, the minimum spot size parameter w_0 can be expressed in terms of two measured beam radii $w(z_1)$ and $w(z_2)$ at locations z_1 and z_2 outside the laser. This expression is

$$w_0 = \frac{\lambda}{\pi}\sqrt{\frac{z_2^2 - z_1^2}{w^2(z_2) - w^2(z_1)}} \quad (4)$$

It should be kept in mind that as per Figure 1, z is measured from the beam waist. The location of the beam waist w_0 is, however, approximately in the center of the laser's casing.

Using Eqs. 3 and 4, the half divergence angle is given by

$$\theta = \sqrt{\frac{w^2(z_2)-w^2(z_1)}{z_2^2-z_1^2}} \qquad (5)$$

An alternative way to estimate the divergence of the beam is by a geometrical approach, as shown in Figure 2, below.

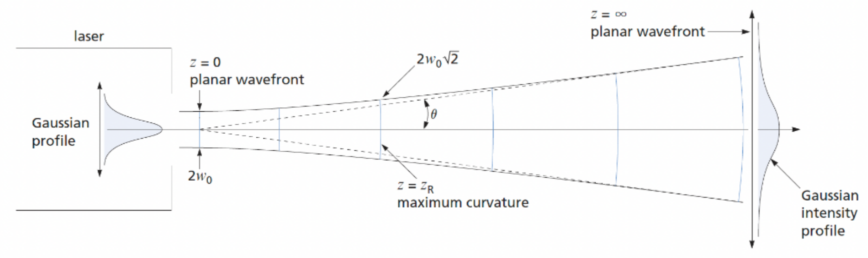

Figure 2. *Divergence angle geometry.*

Using two measured beam radii, $w(z_1)$ and $w(z_2)$, the half divergence angle can be approximated by

$$\theta = \tan^{-1}\left[\frac{w(z_2)-w(z_1)}{z_2-z_1}\right] \qquad (6)$$

<u>Gaussian Beam: Measuring the beam radius</u>

One method to measure the diameter of a laser beam is to measure the power of the beam as a sharp edge is moved to block the beam as shown in Figure 3. Please take due precaution to avoid any diffraction effects from the knife edge by choosing an appropriate position of the power meter or photodiode after the knife edge.

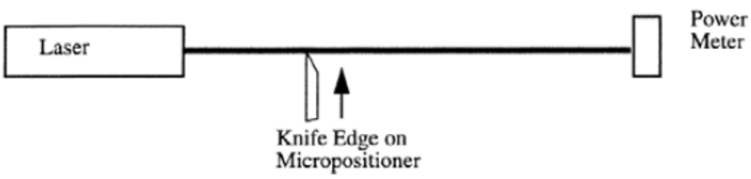

Figure 3. *Setup for Gaussian beam experiment*.

The cross-sectional irradiance distribution for a TEM_{00} wave is shown in Figure 4, below.

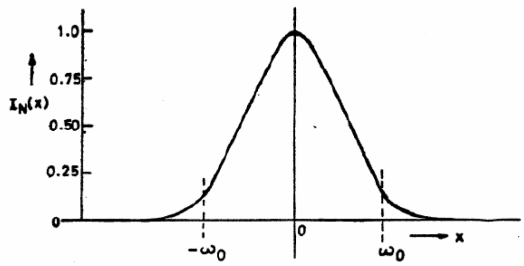

Figure 4. *Irradiance profile of the Gaussian beam*.

Mathematically, the irradiance distribution $I(x, y)$ can be expressed as

$$I(x,y) = \frac{2P_0}{\pi w^2}\exp\left[\frac{-2(x^2+y^2)}{w^2}\right] \qquad (7)$$

Where, the cartesian coordinates x and y are measured from the beam center and are perpendicular to the direction of propagation z. The power transmitted past the sharp edge as the edge is inserted to block the beam is given by

$$P = \int_{-\infty}^{\infty}\int_{a}^{\infty} I(x,y)(dx)dy = \frac{P_0}{2} erfc\left(\frac{a\sqrt{2}}{w}\right) \qquad (8)$$

where, a is the depth of the knife-edge in the beam, P_0 is the power measured by the detector without the knife edge in the beam, and $erfc\left(\frac{a\sqrt{2}}{w}\right)$ is the complementary error function. A sample plot of the expected relative power plot versus knife-edge position is shown in Figure 5, below.

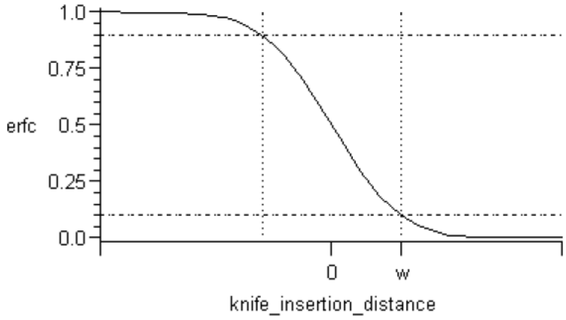

Figure 5. *Relative power distribution as a function of knife-edge position*.

Plotted in Figure 4 are the two positions $-w$ and w, which occur at 90% and 10% of the maximum power. Therefore, the beam diameter $2w$ can be determined by noting the positions of the 10% and 90% relative power points. The choice to use 10% and 90% to determine the beam diameter is arbitrary; the thing that really matters is that the percentages are consistent for all beam diameter measurements.

The same procedure can be repeated by placing a lens (of suitable focal length) after the He-Ne laser. Here, it is important to take the measurement of the beam profile before the focal point as well as after the focal point. Try to take the best data possible for the focal point.

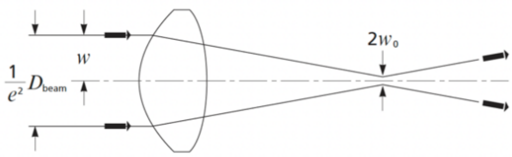

Figure 6. *Concentration of the laser beam using a lens*.

Procedure

(I) Beam Diameter without lens

1. Place the laser on the optical rail (towards one extreme end) and note its position.
2. Place the detector at the other extreme end and align it such that the laser beam hits the center of the detector surface area. Connect the detector output to an external resistor. Connect a multimeter across the resistor to measure the output voltage.
3. Place the knife edge with an attached micrometer stage at a distance from the laser.
4. Move the micrometer stage to introduce the knife edge into the beam path and gradually blocking the beam.

5. Note the detector output voltage at different positions of the knife edge as per Table 1.
6. Obtain a plot as shown in Figure 5 and estimate the beam diameter.

(II) Beam waist and divergence without lens

1. After recording data in the previous procedure, keep the position of the laser and detector unchanged.
2. Place the knife edge at different distances from the laser. At every position record the maximum output of the detector.
3. Then slowly move the knife edge using the micrometer stage such that detector output reads $1/e^2$ of the maximum value recorded in the previous step.
4. Record your data at various distances of the knife edge from laser as given in Table 2.
5. Plot a graph similar to Figure 1 and estimate beam waist and divergence angle.

(III) Beam diameter with lens

1. Keeping the laser and detector position intact, place a lens of known focal length in front of the laser.
2. Repeat steps from 3-5 of procedure (I) and record data as per Table 3.
3. Estimate the beam diameter from a plot as shown in Fig. 5 and compare it with the value obtained without lens.

(IV) Beam waist and divergence with lens

1. Repeat all the steps of procedure (II) in presence of lens and record data as per Table 4.
2. Make sure that the knife edge is placed at positions before and after the focal length of the lens and also at a position close to the focal length in order to obtain a complete graph for correct estimation of beam waist.

Table 1: Beam diameter without lens

Distance of knife edge from laser = ….. cm

Sl. No.	Position of the micrometer with knife edge	Output of Photodetector
1		
2		
..		

Table 2: Beam waist without lens

Sl. No.	Distance of knife edge from laser	Position of the micrometer with knife edge at maximum output (a)	Position of the micrometer with knife edge at $1/e^2$ of maximum output (b)	Beam diameter (b-a)
1				
2				
..				

Table 3: Beam diameter with lens (focal length = …)

Distance of knife edge from laser = ….. cm

Sl. No.	Position of the micrometer with knife edge	Output of Photodetector
1		
2		
..		

Table 4: Beam waist with lens

Sl. No.	Distance of knife edge from laser	Position of the micrometer with knife edge at maximum output (c)	Position of the micrometer with knife edge at $1/e^2$ of maximum output (d)	Beam diameter (d-c)
1				
2				
..				

Graphs:

Plot all the appropriate graphs to obtain the beam parameters without and with lens. In addition, suitably evaluate the error for each reported value.